Konservative Elektrodynamik

$$L=-m\,c\sqrt{u_\beta u^\beta}-qA_\beta u^\beta+\lambda\,q\left(A_\beta u^\beta-V\right)$$

$$\mathscr{L}=\frac{1}{2\mu_0}S_{\alpha\beta}S^{\alpha\beta}+j^\beta A_\beta$$

$$\lambda=1$$
$$m\,\dot{u}_\alpha=q\,\partial_\alpha A_\beta u^\beta$$

$$S_{\alpha\beta}=\partial_\alpha A_\beta$$

$$\partial_\alpha S^{\alpha\beta}=\mu_0\,j^\beta$$

$$\Box A^\beta=\mu_0\,j^\beta$$

$$\left(A^\beta\right)=\left(V,\vec{A}\right)$$

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Konservative Elektrodynamik

$$(A_\beta)=(V,-\vec{A})$$

$$S_{\alpha\beta}=\partial_\alpha A_\beta$$

$$F_{\alpha\beta}=S_{\alpha\beta}-S_{\beta\alpha}$$

$$S_{\beta\alpha}$$

$$\vec{E}=-\nabla V-\frac{\partial\vec{A}}{\partial t}$$

$$\vec{B}=\nabla\times\vec{A}$$

$$(S_{\alpha b})=(\partial_\alpha)\otimes\vec{A}$$

$$\vec{k}=\rho\vec{E}$$
$$\vec{K}=q\vec{E}$$

$$\vec{k}=\rho\vec{v}\times\vec{B}$$
$$\vec{K}=i\vec{l}\times\vec{B}$$

$$\vec{k}=\rho(u^\alpha)(S_{\alpha b})$$
$$\vec{K}=i\vec{A}$$

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Konservative Elektrodynamik

Klassisch (v « c): $L=\frac{1}{2}m v^2-q(V-\vec{A}\vec{v})+\lambda_q q(V-\vec{A}\vec{v}-V_0)$

Relativistisch (v < c): $L=-m c^2-qA_\beta u^\beta-\lambda_m m(u_\beta u^\beta-c^2)+\lambda_q q(A_\beta u^\beta-V_0)$

λ_m, λ_q … Lagrange-Multiplikatoren

Nebenbedingungen:

$\frac{\partial L}{\partial \lambda_m}=0 \Rightarrow u_\beta u^\beta=c^2$ $\qquad \lambda_m=\frac{1}{2}$

$\frac{\partial L}{\partial \lambda_q}=0 \Rightarrow A_\beta u^\beta=V_0$ $\qquad \lambda_q=0$ Maxwellsche Elektrodynamik

$\lambda_q=1$ KED

$\lambda_q=1$:

$$L=-\frac{1}{2}m u_\beta u^\beta-\left(\frac{1}{2}m c^2+q A_\beta u^\beta\right)=T-U$$

$$E=-\frac{1}{2}m u_\beta u^\beta+\left(\frac{1}{2}m c^2+q A_\beta u^\beta\right)=T+U$$

$T \stackrel{\text{def}}{=} -\frac{1}{2}m u_\beta u^\beta$ Kinetische Energie

$U \stackrel{\text{def}}{=} \frac{1}{2}m c^2+q A_\beta u^\beta$ Potenzielle Energie

Dieser Term fehlt hier in der Maxwellschen Elektrodynamik!

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