The relativistic asynchronous machine - an application of General Theory of Relativity in electrical engineering?

Up to now applications of General Theory of Relativity are known in astrophysics like the redshift or the slow run of resting clocks in a gravitational field.
Unusual seems to be an application in electrical engineering. A suitable construction of a relativistic version of the three-phase current asynchronous motor could be used as an instrument to prove the equivalence principle and theoretically makes travels to the future possible.

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1 Equivalence principle

There is now doubt that General Theory of Relativity (GR) is beside quantum theory the most important basis of modern physics. It is founded on the so called equivalence principle which can be formulated according to experimental experience in following different ways:

- There is no gravity relative to free falling reference bodies.
- Gravitational mass and inertial mass are equal to each other.

This leads to some conclusions:

- Gravitational force is equal to inertial force.
- All bodies fall with the same speed.
- A reference frame connected to a free falling body ist a local inertial frame (IF).
- In the local IF valid physical laws are laws without gravity based on Special Theory of Relativity (SR).
- By change from local IF to any moved frames (by means of coordinate transformations) physical laws are generally usable in accelerated frames or in gravitational fields.
- Technical generated inertial forces (e.g. the centrifugal force in a motor) can be considered as forces of an artificial gravitational field.

In this article the last conclusion should be of special interest. Is it on principle possible, to generate an artificial gravitational field by means of a “relativistic” motor? This opportunity seems not to be excluded, because electrical motors are described by Maxwell's equations. But in present machines these equations are only used in an incomplete form. In a relativistic machine the complete equations would be necessary, because they are inherently relativistic. Concerning to GR they can be formulated for accelerated frames. So we must search for a suitable construction. A promising candidate could be the asynchronous machine with short circuit rotor. Therefore this machine will be briefly discussed in the following chapter.

2 Asynchronous machine

The asynchronous machine is an electrical machine which operates with multi-phase alternating current to create a rotating magnetic field. Here Maxwell's equations have the form

$$\vec{E} = -\,\mathrm{grad}V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \mathrm{curl}\,\vec{A},$$

$$\mathrm{curl}\,\vec{B} = \mu_0 \,\vec{j},$$

i.e. by using low frequency currents the displacement current is neglected. It is v ≪ c and the circumfence velocity of the air gap field ≪ c (quasi-stationary field). Maxwell's equations used in this way imply the validity of Galilei transformation

$$t = t',$$

$$\varphi = \omega t' + \varphi'.$$

The machine can be used in three different operating modes. These are

- Motoring mode
- Generating mode
- Braking mode

Here an interest is taken in motor application.
The machine contains a resting part (stator) which consists of a stack of iron laminations shaping a thick-walled tubular core. Inside there are axial slots which contains conducting bars. At the ends they are normaly connected to three windings placed in an spatial angular of 120° to each other. These windings are fed with alternating currents which have a time angular difference of 120°. So the stator becomes an electromagnet with a rotating magnetic field.
The rotating field gets through a cylinder-shaped stack of laminations called rotor mounted in the stator for rotary motion. Rotor and stator are separated only by a thin air-gap. In the air-gap the rotating field vector has a radial direction. As a consequence of the existing windings it consists of a fundamental wave and an infinite number of harmonics. The behaviour of the machine is determined by the fundamental wave while the harmonics affect a disturbing influence on it. The shape of the fundamental wave is

$$B_r(\varphi, t) = m\,\frac{B_0}{2}\cos(\omega_0 t - p\varphi), \quad \mathrm{m}\geq 3, \quad \text{natural number.} \qquad (2.1)$$

The angular velocity of the field is determined by the quotient of frequency ω_0 to number of pole pairs p. For a three-phase current system m is equal to three. With regard to the relativistic machine the four-phase system is interesting because by connecting the phases 1-3 and 2-4 to winding pairs its behaviour is equal to a special two-phase system. Only two windings are placed in an spatial angular of 90° to each other. The alternating currents have a time angular difference of 90°.
The rotor also carries conducting bars lying in axial slots. Manufactured as squirrel-cage rotor or short circuit rotor the bars are connected at the ends with short circuit rings. A current is generated in the bars by induction, by which the rotor becomes an electromagnet. As a result the rotor will be pulled with the stator rotating field and accelerates until the machine torque balances the load torque. Then the rotor speed ω is constant and remains lower than the stator speed in any case. The machine slip s is defined as

$$s = 1 - \frac{\omega}{\omega_0}.$$

Therefore the denotation “asynchronous”.

3 Maxwell's equations

Maxwell's equations read in complete form

$$\vec{E} = -\operatorname{grad} V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \operatorname{curl} \vec{A} \quad (3.1),$$

$$\operatorname{curl} \vec{B} - \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j}, \ \operatorname{div} \vec{E} = \mu_0 \rho \quad (3.2).$$

In this form this equations can be used in inertial frames and with curved spatial coordinates. For use in accelerated frames with reference to GR it is usual to write them in a covariant form by means of tensors. Then physical quantities appears as 4-component vectors (first rank tensors) or tensors of second rank. It is suitable that the quantities E, V, t and ρ get the same unit as B, A, x and j. So for instance the electric field strength will be measured in T (Tesla) instead of V/m. The scalar potential V and the vector potential A are combined to a electromagnetic potential in form of a (contravariant) 4-vector:

$$\left(A^{\alpha}\right) = \left(V, A_x, A_y, A_z\right).$$

Here cartesian coordinates were used. According to that there is a 4-vector current density

$$\left(j^{\alpha}\right) = \left(\rho, j_x, j_y, j_z\right),$$

and a 4-vector for time and space

$$\left(x^{\alpha}\right) = (t, x, y, z).$$

The components of contavariant vectors are indicated by superscript indices. We obtain the covariant vector by multiplication with the second rank covariant tensor $g_{\alpha\beta}$:

$$A_{\alpha} = g_{\alpha\beta} A^{\beta}.$$

In SR the metric tensor reads in cartesian coordinates (written as a matrix):

$$\left(g_{\alpha\beta}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

With reference to GR the metric tensor in general depends on time and space and has the caracter of a gravitational or inertial potential. It has the symmetry property $g_{\alpha\beta} = g_{\beta\alpha}$.

Because relativity provides time as additional fourth dimension an remarkable view of Maxwell's equations results: The electric field E and the magnetic field B, two basic forces of nature, are usually represented as 3-dimensional vectors. In four dimensions both fields are unified in Maxwell's theory to one electromagnetic field:

$$\left(F_{\alpha\beta}\right) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}.$$

This formulation shows that the electric and magnetic field fundamentally are no different physical quantities- they have equal rights. Now equations (3.1) can be combined to

$$F_{\alpha\beta} = \frac{\partial A_\beta}{\partial x^\alpha} - \frac{\partial A_\alpha}{\partial x^\beta} \qquad (3.3).$$

Accordingly equations (3.2) can be combined to

$$\frac{\partial}{\partial x^\alpha}\left(\sqrt{g} F^{\alpha\beta}\right) = \mu_0 \sqrt{g} j^\beta \qquad (3.4),$$

where $g = -\det\left(g_{\alpha\beta}\right)$. It is further on $F^{\alpha\beta} = g^{\alpha\chi} g^{\beta\delta} F_{\chi\delta}$.

In this generalized Maxwell equations implicitly the metric tensor appears. This gives us a hint to the opportunity how to discribe a machine which creates a rotary motion by means of an electromagnetic field. This yields a metric causing a centrifugal force. This facts are typical for rotating field machines like the asynchronous motor.

4 Relativistic asynchronous machine

To construct the machine we choose a device from high frequency technology, the cylindric cavity resonator. It is axisymmetrical and here Maxwell's equations are valid in their complete form. Now the high frequent electromagnetic field of the mode TM_{110} will be stimulated. This mode is determined by Maxwell's equations and the boundary conditions. It exists in two degenerated types at the same resonance frequency i.e. both types can be independently stimulated and than they interfere (with J_1 = Bessel function of first order, η = normalized radius):

Mode 1:

$$E_z(r, \varphi, t) = E_0 J_1(\eta)\cos\omega_0 t \cos\varphi ,$$

$$B_r(r, \varphi, t) = E_0 \frac{J_1(\eta)}{\eta}\sin\omega_0 t \sin\varphi ,$$

$$B_\varphi(r, \varphi, t) = E_0 J'_1(\eta)\sin\omega_0 t \cos\varphi .$$

Mode 2:

$$E_z(r, \varphi, t) = E_0 J_1(\eta)\cos\omega_0 t \sin\varphi ,$$

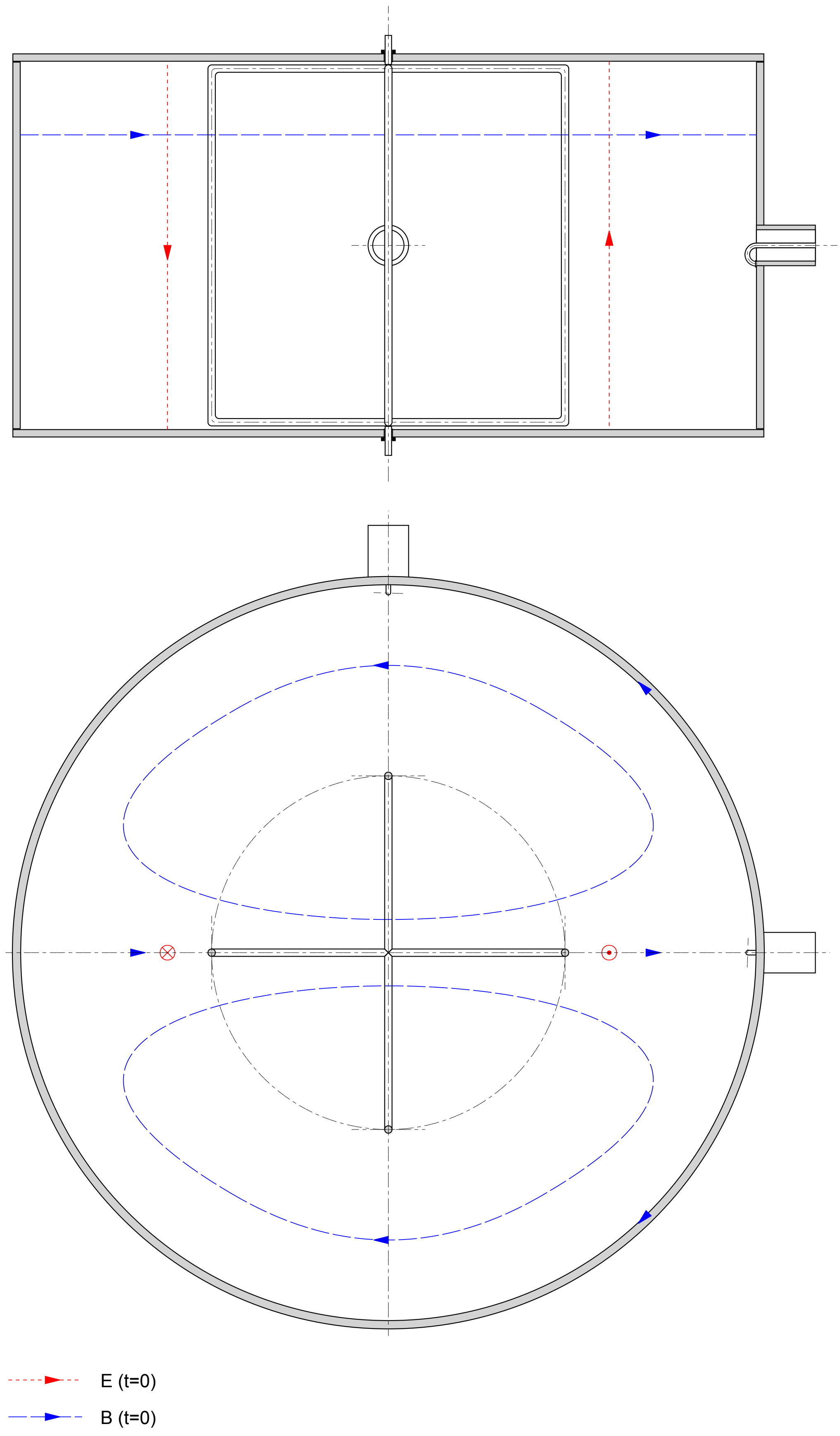
$$B_r(r, \varphi, t) = -E_0 \frac{J_1(\eta)}{\eta}\sin\omega_0 t \cos\varphi ,$$

$$B_\varphi(r, \varphi, t) = E_0 J'_1(\eta)\sin\omega_0 t \sin\varphi .$$

The resonance frequency is given by

$$\omega_0 = \eta_{11} \frac{c}{R} .$$

($\eta_{11} \approx 3.8317$, first zero point of Bessel function of first order, c=speed of light, R=resonator radius). The field structures of both modes are identical, but they are displaced from each other by an angular of 90°. Therefore we have the case of the two-phase machine as we have seen in chapter 2. The modes can be stimulated by inductive coupling via two coaxial lines. They are soldered on holes drilled with an angular difference of 90° into the cylinder case.


E (t=0)
B (t=0)

If the second mode is stimulated with a time delay by T/4 to the first, both modes interfere to an electromagnetic rotating field:

$$E_z(r,\ \varphi,\ t) = E_0 J_1(\eta)\big(\cos\omega_0 t\cos\varphi + \sin\omega_0 t\sin\varphi\big) = E_0 J_1(\eta)\cos\big(\omega_0 t - \varphi\big)\,,$$

$$B_r(r,\ \varphi,\ t) = E_0\,\frac{J_1(\eta)}{\eta}\big(\sin\omega_0 t\sin\varphi + \cos\omega_0 t\cos\varphi\big) = E_0\,\frac{J_1(\eta)}{\eta}\cos\big(\omega_0 t - \varphi\big)\,,$$

$$B_\varphi(r,\ \varphi,\ t) = E_0\,J'_1\,(\eta)\big(\sin\omega_0 t\cos\varphi - \cos\omega_0 t\sin\varphi\big) = E_0\,J'_1\,(\eta)\sin\big(\omega_0 t - \varphi\big)\,.$$

These field components result from the electromagnetic potential

$$A_z(r,\ \varphi,\ t) = -\frac{E_0}{\omega_0}\,J_1(\eta)\sin\big(\omega_0 t - \varphi\big) + \textit{gauge term} \qquad (4.1).$$

It solves the wave equation

$$\Delta A_z = \frac{\partial^2 A_z}{\partial t^2} \qquad (4.2).$$

The field rotates with a speed equal to the mode resonance frequency (n_0=f_0). The structure of the magnetic field is similar to that of the conventional asynchronous machine (number of pole pairs p=1). Contrary to the conventional asynchronous machine the magnetic field is guided by the metallic cylinder case instead of an iron core. The electric field vector is perpendicular to the magnetic field vector and is parallel to cylinder axis.
At the radius $\eta_0 \approx 1.8412$ $J_1'(\eta)$ vanishes and we have

$$B_r\big(r_0,\ \varphi,\ t\big) = E_0\,\frac{J_1(\eta_0)}{\eta_0}\cos\big(\omega_0 t - \varphi\big)\,.$$

Except the magnitude this equation describes a field similar to the air-gap field [equation (2.1) with p=1]. In contrast to the conventional asynchronous machine the field here only consists of a fundamental wave.
The bottom and top of the resonator have a bearing to mount the short circuit rotor. It consists of two rectangular conducting loops with equal size. They are crossed in an angular of 90° and solded to each other. Then the rotor is shaped like a crossed frame antenna (similar to those used for radio bearing sometimes) and is penetrated by the rotating field.

To develop a useful mathematic model we assume that the rotating field is hardly disturbed by the rotor.
The field at $\eta_0 \approx 1.8412$ is a component parallel to the cylinder axis

$$E_z\big(r_0,\ \varphi,\ t\big) = E_0 J_1\big(\eta_0\big)\cos\big(\omega_0 t - \varphi\big)\,.$$

Therefore a voltage drops over the conductors which are parallel to the z-axis. Because of Ohm's law and high frequencies for these conductors the impedance yields to

$$\underline{Z} = R_\mathrm{V} + \mathrm{i}\omega_0 L \quad \text{with} \quad R_\mathrm{V} = \omega_0 L = \frac{l}{\pi d}\sqrt{\frac{\mu_0\,\omega_0}{2\kappa}}$$

(l=length, d=diameter and κ=conductivity of the conductor).
With rising frequency the resistance of the short circuit rotor increases what is known as skin-effect from the conventional machines.
In the magnetic field B_r the currents flowing through the conductors generate a torque based on Lorentz forces. As a result the rotor will be pulled by the stator rotating field and starts rotating. It accelerates if the machine torque

$$M_\mathrm{A} = \frac{\big[lE_0 J_1(\eta_0)\big]^2}{\omega_0 R_\mathrm{V}}$$

is greater than the load torque (caused at least by bearing friction). As a result of decreasing speed difference between rotating field and rotor induction and also machine torque decreases until it balances load torque. Each load torque causes a determined constant speed. For these stable operation modes and speeds the torque delivered by the rotor can be computed if the field strength with view from the rotor is known. Hence a suitable coordinate transformation is necessary between the resting reference frame (the inertial frame) and the rotating reference frame.

First we give the tensor descriptions of the electromagnetic and metric field in the resting frame in cylinder coordinates:

$$\left(F_{\alpha\beta}\right)=\begin{pmatrix} 0 & 0 & 0 & E_z \\ 0 & 0 & 0 & B_\varphi \\ 0 & 0 & 0 & -\frac{r}{R}B_r \\ -E_z & -B_\varphi & \frac{r}{R}B_r & 0 \end{pmatrix},$$

$$\left(g_{\alpha\beta}\right)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\left(\frac{r}{R}\right)^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (4.3).$$

Now we make a statement for a general transformation V between the resting frame S and the rotating frame S':

$$\begin{aligned} t &= l\,t' + m\,\varphi' \\ r &= r' \\ \varphi &= n\,t' + o\,\varphi' \\ z &= z' \end{aligned} \qquad (4.4).$$

The coordinates t' and φ' nevertheless have not to be interpreted as “time” and “angle” (*General* Theory of Relativity).

In tensor formulation the transformation V of the 4-location vector yields $x^\alpha = V^\alpha{}_\beta\, x'^\beta$ with

$$\left(V^\alpha{}_\beta\right)=\begin{pmatrix} 1 & 0 & \mathrm{m} & 0 \\ 0 & 1 & 0 & 0 \\ \mathrm{n} & 0 & \mathrm{o} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The electromagnetic and metric field than transform as follows:

$$F'_{\alpha\beta} = V^\chi{}_\alpha V^\delta{}_\beta F_{\chi\delta},$$

$$g'_{\alpha\beta} = V^\chi{}_\alpha V^\delta{}_\beta g_{\chi\delta}.$$

Now the problem is to determine the unknown transformation parameters l, m, n and o. To that we fix the electromagnetic potential in a suitable way. We put the gauge term in equation (4.1) to zero and postulate that the potential $(A_\alpha) = (0, 0, 0, -A_z)$ transforms like a tensor of first rank:

$$A'_\alpha = V^\beta{}_\alpha A_\beta \quad \text{,i. e.} \quad A'_z = A_z \qquad (4.5).$$

For any curved coordinates or reference frames with Maxwell's equations (3.3), (3.4) we obtain the generalization of the wave equation (4.2):

$$\frac{\partial}{\partial x^{\alpha}}\left(\sqrt{g}g^{\alpha\beta}\frac{\partial A_z}{\partial x^{\beta}}\right) = 0 \, .$$

Thereby the $g^{\alpha\beta}$ results from the relationship $g^{\alpha\nu}g_{\nu\beta} = \delta^{\alpha}_{\beta}$.

To perform the change from the resting to the rotating frame we replace the unprimed quantites by primed in the wave equation. Then the solving of the wave equation gives us an opportunity to find the searched transform parameters. In the wave equation we substitude the $g'^{\alpha\beta}$ which still contains the unkown parameters. The statement for the substituted A'_z results from (4.5):

$$A'_z\,(r,\,\varphi',\,t') = -\frac{E'_0}{\omega'_0}J_1(\eta)\sin\left(\omega'_0\,t' - \varphi'\right) = -\frac{E_0}{\omega_0}J_1(\eta)\sin\left(\omega_0 t - \varphi\right) \qquad (4.6).$$

Obviously this statement is only possible if the wave equation can be solved with separation so that we get Bessel's differential equation of first order. But this implies transformation parameters independent from time and space. Because then we obtain two equations from Bessel's differential equation with a new normalization of the radius on the one hand and with a coefficient comparison on the other to determine the parameters:

$$\eta = \frac{r}{R}\,\frac{n+o\,\omega'_0\frac{R}{c}}{lo-mn} \qquad (4.7),$$

$$1 = \frac{l+m\,\omega'_0\frac{R}{c}}{lo-mn} \qquad (4.8).$$

Because the first zero point of the Bessel function of first order is always constant (η_{11}) with (4.7), (4.8) a relation yields between ω_0 and ω'_0:

$$\omega_0\,\frac{R}{c} = \frac{n+o\,\omega'_0\frac{R}{c}}{l+m\,\omega'_0\frac{R}{c}} \qquad (4.9).$$

From (4.6) we have the two relationships:

$$\frac{E'_0}{\omega'_0} = \frac{E_0}{\omega_0} \qquad (4.10),$$

$$\omega'_0\,t' - \varphi' = \omega_0 t - \varphi \quad (4.11).$$

Also we have to consider that the rotor is moving relative to the stator with the angular velocity ω [see (4.4)]:

$$\varphi' = const. \quad \Rightarrow \quad \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega \quad \Rightarrow \quad \omega = \frac{n}{l} \quad (4.12),$$

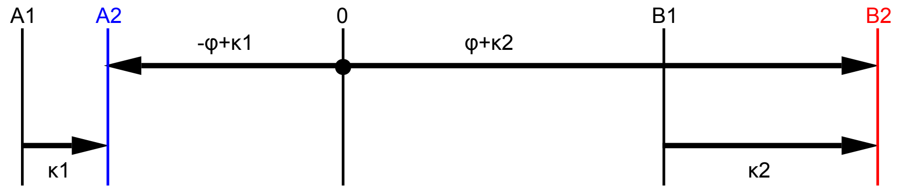
$$\varphi = const. \quad \Rightarrow \quad \frac{\mathrm{d}\varphi'}{\mathrm{d}t'} = -\,\omega \quad \Rightarrow \quad l = o \quad (4.13).$$

With (4.9), (4.12) and (4.13) we obtain:

$$\frac{\omega'_0}{\omega_0} = \frac{1-v}{1-\omega_0\frac{m}{l}} \qquad (4.14)$$

with $v = \frac{\omega}{\omega_0} = \frac{n}{f_0}$.

The constant m/l results from the following consideration: The oscillating field with the frequency ω_0 is a standing wave, which can be divided in a clockwise rotating wave (frequency ω_0) and a counter-clockwise rotating wave (frequency ω_0).



When the rotor moves clockwise with angular velocity ω (see fig.) rotor and counter-clockwise rotating field meet at place A_2 and time

$$t_1 = \frac{\kappa_1}{\omega} = \frac{-\varphi+\kappa_1}{-\omega_0}\,.$$

On the other hand the clockwise rotating field passes the rotor at place B_2 and time

$$t_2 = \frac{\kappa_2}{\omega} = \frac{\varphi+\kappa_2}{\omega_0}\,.$$

As a result we have:

$$\kappa_1 = \frac{\varphi v}{1+v}\,,$$

$$\kappa_2 = \frac{\varphi v}{1-v}\,.$$

We define the quotient $s = \frac{t_1}{t_2}$ and get

$$s = \frac{1-v}{1+v} = \frac{\omega_0(1-v)}{\omega_0(1+v)} = \frac{\omega'_0}{\omega_0} \qquad (4.15).$$

A comparison of (4.15) with (4.14) results in

$$\frac{m}{l} = -\frac{v}{\omega_0} \qquad (4.16).$$

With (4.8), (4.12), (4.13), (4.15) and (4.16) we obtain the searched transformation

$$\left(V^{\alpha}{}_{\beta}\right) = \begin{pmatrix} \frac{1}{1+v} & 0 & -\frac{v}{\eta_{11}(1+v)} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\eta_{11} v}{1+v} & 0 & \frac{1}{1+v} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (4.17).$$

Equation (4.15) also can be written as

$$1 = \frac{s+v}{1-sv}\,.$$

With s=tanσ and v=tanα we obtain for the *relativistic addition of angular velocities*:

$$\frac{\pi}{4} = \sigma + \alpha\,.$$

The quantity s is the *slip* of the relativistic machine. It could also be named as redshift factor because the observer on the rotor sees a red shifted resonance frequency. According to (4.10) it is also $s = \frac{E'_0}{E_0}$. This are relationships which are already known

from the conventional asynchronous machine.
In the special case that the rotor speed is very small against the resonance frequency of the resonator ($v \ll 1$) we obtain the approximation

$$t \approx t' - \frac{v}{\omega_0} \varphi' ,$$

$$\varphi \approx \omega t' + \varphi' .$$

The transformation (4.17) can be graphically presented in a spacetime diagram with the coordinates t and φ (see figure). When the axes φ, φ', t and t' are standardised (ω_0 is included in the standardisation of t and t') then we have

$$t = \frac{1}{1+v} t' - \frac{v}{1+v} \varphi' ,$$

$$\varphi = \frac{v}{1+v} t' + \frac{1}{1+v} \varphi' .$$

The standardised synchronous angular velocity ω_0 is presented by a bisecting line in the reference frame S. For the phase term (4.11) we obtain st'-φ'=t-φ. With v=tanα we have

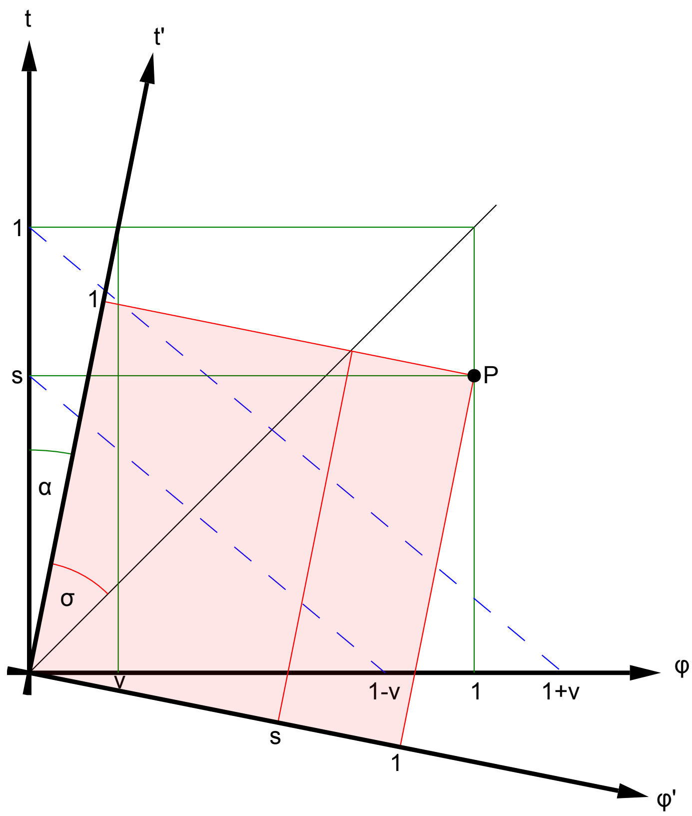
$$t = \frac{\cos\alpha}{\cos\alpha+\sin\alpha} t' - \frac{\sin\alpha}{\cos\alpha+\sin\alpha} \varphi' ,$$

$$\varphi = \frac{\sin\alpha}{\cos\alpha+\sin\alpha} t' + \frac{\cos\alpha}{\cos\alpha+\sin\alpha} \varphi' .$$

This means that the spacetime in transition from S to S' is rotated by an angle α around the origin and is shrinking by a factor (cosα+sinα). To construct the spacetime (t',φ') geometrically the axes t and φ will be arbitrarily scaled. Now the slip s will be marked on the t-axis by means of the theorem on intersecting lines. Considering the event P(s,1) in S we obtain t'=1 and φ'=1 in S' according to the transformation formula. Note that the scaling of the t'-axis and φ'-axis is smaller. Here the value s for the standardised angular velocity of the rotating field can now be read from the φ'-axis.

The frame S' can also be called “virtual spacetime”. The coordinates t' and φ' are exactly determined by (4.17) - one can “see” them (with mathematical calculations) but can not measure them directly (similar to a reflected image in a mirror that one can only see but can not be measured as a real image). As mentioned above they also can not be interpreted as “time” and “angle” anymore. However measurable is and a physical meaning has the space time interval given by the first fundamental form

$$\mathrm{d}s^2 = c^2 \mathrm{d}\tau^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = g'_{\mu\nu} \mathrm{d}x'^\mu \mathrm{d}x'^\nu .$$



Now we can give the searched fields in the reference frame S':

$$\left(F'_{\alpha\beta}\right)=\begin{pmatrix} 0 & 0 & 0 & sE_z \\ 0 & 0 & 0 & B_\varphi \\ 0 & 0 & 0 & -\frac{r}{R}B_r \\ -sE_z & -B_\varphi & \frac{r}{R}B_r & 0 \end{pmatrix} \qquad (4.18),$$

$$\left(g'_{\alpha\beta}\right)=\begin{pmatrix} g'_{00} & 0 & g'_{02} & 0 \\ 0 & -1 & 0 & 0 \\ g'_{20} & 0 & g'_{22} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

with

$$g'_{00} = \frac{1-(v\eta)^2}{(1+v)^2},$$

$$g'_{02} = g'_{20} = -\frac{v\left(1+\eta^2\right)}{\eta_{11}(1+v)^2},$$

$$g'_{22} = \frac{v^2-\eta^2}{\eta_{11}{}^2(1+v)^2}.$$

In case of the resting rotor (v=0) the metric components reduces to those in (4.3).
Now the centrifugal force can be computed which is acting on the moving rotor. To an observer resting on the rotor it appears like a gravitational force with a radial direction away from the center. In GR these forces are described by Christoffel's symbols:

$$\Gamma^{\kappa}_{\mu\nu} = \frac{g^{\kappa\lambda}}{2}\left(\frac{\partial g_{\nu\lambda}}{\partial x^{\mu}} + \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} - \frac{\partial g_{\nu\mu}}{\partial x^{\lambda}}\right).$$

These symbols are the “analogy” to the equations (3.3) of electrodynamics. The only symbol we need for the centrifugal force is the radial component Γ^{1}_{00} . Additionally we still need the 4-velocity u' of the observer on the rotor. Starting from the first fundamental form

$$\mathrm{d}s^2 = g'_{\mu\nu}\mathrm{d}x'^{\mu}\mathrm{d}x'^{\nu}$$

we obtain $(u'^{\mu}) = \left(\frac{c}{\sqrt{g'_{00}}},\ 0,\ 0,\ 0\right)$. With the motion equations

$$\frac{\mathrm{d}u^{\kappa}}{\mathrm{d}\tau} = -\Gamma^{\kappa}_{\mu\nu}u^{\mu}u^{\nu}$$

then for the acceleration follows

$$b_r = -\Gamma^{1}_{00}\left(u'^{0}\right)^2 = \gamma^2\omega^2 r$$

with $\gamma = \frac{1}{\sqrt{1-\left(\frac{\omega r}{c}\right)^2}}$. For ωr≪c we obtain the already known expression from classical mechanics.

The 4-velocity of the observer in the resting reference frame is given by

$$u^{\nu} = V^{\nu}{}_{\mu}\,u'^{\mu} = \left(u_t,\ u_r,\ \frac{R}{r}u_{\varphi},\ u_z\right) = \gamma(c,\ 0,\ \omega R,\ 0)\,.$$

According to SR a body which gets loose from the moving rotor will move along a straight line with the constant 4-velocity

$$u_t = \gamma c, \quad u_{\varphi} = \gamma\omega r = \gamma v_{\varphi}\,.$$

From the condition $\omega r < c$ we obtain an upper limit for the rotor speed:

$$v < v_{\max} = \frac{1}{\eta_0} \approx 0.543$$

Hence we get with (4.15) a result for the lowest possible slip

$$s_{\min} = \frac{\eta_0-1}{\eta_0+1} \approx 0.3.$$

In comparison to that a conventional induction motor has a typical slip of about 0.01.

With the electromagnetic field (4.18) we obtain the searched relationship between torque and speed in the inertial frame

$$M(\mathrm{n}) = M_A \sqrt{1 - \left(\frac{n}{f_0}\right)^2} \;.$$

Here we have used a modification of Ohm's law

$$j'_z = \frac{\kappa E'_z}{\sqrt{g'_{00}(r_0)}} \;.$$

If the machine's behaviour is like the relationship M(n) another proof of the equivalence principle and hence GR is furnished.

The expressions for the fields still can be extended to all pole-pair-numbers p. Then we have

$$A'_z\,(r,\, \varphi',\, t') = -\frac{E'_{p1}}{\omega'_{p1}}\, J_p(\eta)\sin\!\left(\omega'_{p1}\, t' - p\,\varphi'\right)$$

with $s = \frac{1-pv}{1+pv}$ and $\omega'_{p1} = s\eta_{p1}\,\frac{c}{R}$. p is an integer and discrete eigenvalue. Its sign decides in which direction the rotating field moves. It is obvious also to consider the slip s as an eigenvalue which can vary continuously between 1 (motor standstill) and 0. Because of an infinite number of eigenvalues p we also obtain an infinite number of "Eigenmetrics"

$$g'_{00} = \frac{1-(v\eta)^2}{(1+pv)^2}\,,$$

$$g'_{02} = g'_{20} = -\frac{p}{\eta_{p1}}\,\frac{pv\left[1+\left(\frac{\eta}{p}\right)^2\right]}{(1+pv)^2}\,,$$

$$g'_{22} = \frac{p^2}{{\eta_{\mathrm{p}1}}^2}\,\frac{(pv)^2-\left(\frac{\eta}{p}\right)^2}{(1+pv)^2}\,.$$

and "Eigentransformations"

$$\left(V^{\alpha}{}_{\beta}\right) = \begin{pmatrix} \frac{1}{1+pv} & 0 & -\frac{p^2 v}{\eta_{p1}\left(1+pv\right)} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\eta_{\mathrm{p}1} v}{1+pv} & 0 & \frac{1}{1+pv} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

5 Coriolis effect

To discuss the Coriolis effect we set up the motion equations. The Christoffel's symbols are:

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{v^2}{1+v^2}\frac{1}{r},$$
$$\Gamma^0_{12} = \Gamma^0_{21} = \frac{v}{1+v^2}\frac{1}{\eta_{11}}\frac{1}{r},$$
$$\Gamma^1_{00} = -\frac{1}{(1+v)^2}\frac{\omega^2}{c^2}r,$$
$$\Gamma^1_{02} = \Gamma^1_{20} = -\frac{1}{(1+v)^2}\frac{\omega}{c}\frac{r}{R},$$
$$\Gamma^1_{22} = -\frac{1}{(1+v)^2}\frac{r}{R^2},$$
$$\Gamma^2_{01} = \Gamma^2_{10} = \frac{v}{1+v^2}\eta_{11}\frac{1}{r},$$
$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{1+v^2}\frac{1}{r}.$$

All remaining $\Gamma^\kappa_{\mu\nu}$ are equal zero. If $\omega=0$ (rotor standstill) the case of polar coordinates remains:

$$\Gamma^1_{22} = -\frac{r}{R^2},$$
$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}.$$

As a result we get the motion equations

$$\frac{du'^0}{d\tau} = -2\Gamma^0_{01}u'^0u'^1 - 2\Gamma^0_{12}u'^1u'^2,$$
$$\frac{du'^1}{d\tau} = -\Gamma^1_{00}\left(u'^0\right)^2 - 2\Gamma^1_{02}u'^0u'^2 - \Gamma^1_{22}\left(u'^2\right)^2 \qquad (5.1),$$
$$\frac{du'^2}{d\tau} = -2\Gamma^2_{01}u'^0u'^1 - 2\Gamma^2_{12}u'^1u'^2,$$
$$\frac{du'^3}{d\tau} = 0.$$

Also essential for a mass body ($ds > 0$) is the side condition

$$c^2 = g'_{00}\left(u'^0\right)^2 + g'_{11}\left(u'^1\right)^2 + 2g'_{02}u'^0u'^2 + g'_{22}\left(u'^2\right)^2 + g'_{33}\left(u'^3\right)^2 \qquad (5.2).$$

We consider a body which departs from the center with a constant velocity υ_r. In the resting frame S the body then moves free falling ($b_r = 0$) along a straight line (radius), which represents a geodetic line. The 4-velocity is

$$u^\nu = \gamma(c,\ \upsilon_r,\ 0,\ 0)$$

with $\gamma = \frac{1}{\sqrt{1-\left(\frac{\upsilon_r}{c}\right)^2}}$. Because of $u^\nu = V^\nu{}_\mu u'^\mu$ then in the rotating frame S' must be:

$u'^0 = k\gamma c\,,$
$u'^1 = \gamma \upsilon_r\,,$
$u'^2 = -k\,\gamma\omega R\,,$
$u'^3 = 0\,,$

with $k = \dfrac{1+v}{1+v^2}$. The integration $x'^\mu = \int u'^\mu d\tau$ yields

$t' = k\gamma c\tau\,,$
$r = \gamma\upsilon_r\tau\,,$
$\varphi' = -k\,\gamma\omega R\tau\,,$
$z = 0\,.$

After an affine transformation geodetic lines again become geodetic lines. Then the trajectory in S' must also be a geodetic line, as one can check out by insertion in (5.1) and (5.2)
($b'_{t'} = b'_r = b'_{\varphi'} = 0$). Elimination of τ yields

$$r = -\frac{\upsilon_r}{k\omega R}\varphi' \qquad (5.3).$$

The geodetic line is an *Archimedean Spiral*.

To determine the trajectory of a massless photon ($ds = 0$) we have to choose another parameter (λ instead of τ) in (5.1). Now the side condition is

$$0 = g'_{00}\left(u'^0\right)^2 + g'_{11}\left(u'^1\right)^2 + 2g'_{02}\,u'^0\,u'^2 + g'_{22}\left(u'^2\right)^2 + g'_{33}\left(u'^3\right)^2 .$$

As 4-velocity in S we put:

$$u^\nu = (c,\ c,\ 0,\ 0)\,.$$

Then in the rotating frame S' is

$u'^0 = kc\,,$
$u'^1 = c\,,$
$u'^2 = -k\,\omega R\,,$
$u'^3 = 0\,.$

The integration $x'^\mu = \int u'^\mu d\lambda$ yields

$t' = kc\lambda\,,$
$r = c\lambda\,,$
$\varphi' = -k\,\omega R\lambda\,,$
$z = 0\,.$

Analogous to (5.3) we get

$$r = -\frac{c}{k\omega R}\varphi' \qquad (5.4).$$

A particle beam radiating away from the center would be bent against the rotor's motion like the water jet of a rotating lawn-sprinkler. Especially (5.4) describes a *light bending*.

6 Outlook

Is it generally possible to build such a machine? The challenges in technology which we are faced are huge. The problem is that the relativistic machine shows extreme operation data: Imagine we have a machine with a few meters in diameter. Such a machine would have a resonance frequency (= synchronous speed) in the range of 10^6-10^7 1/s. Theoretically we would come to very high speeds. The contrarily point is that the rotor only has a limited solidity - it would inevitably tear itself because of the large centrifugal force. Add to this that because of the high resonance frequency the machine yields an extremly weak, even microscopically small torque. The torque could be raised by an increase of the field strength in the resonator. This opportunity is restricted by 1. the punch through field strength which is a finite quantity and 2. a high loss of power which appears in the starting rotor and leads to destruction.
Besides this difficulties we could examine interesting physical effects with such a machine:
Time travel. The machine could be "alienated" for travelling to the future. For the observer on the rotor is $d\varphi'=0$. Then a relationship results between the observer's proper time and the time of an observer resting in the inertial frame (i. e. beside the stator):

$$d\tau = \sqrt{1-\left(\frac{\omega r}{c}\right)^2}\,dt \,.$$

There is a difference between the times of both observers. This is appropriate to the term "asynchronous machine": The machine's asynchronous behaviour is not only relative to speed but also to the time flow of clocks. The observer on the rotor grows older more slowly than the resting. The higher the speed or the distance from the center the stronger this effect will be.

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