The relativistic Quantum Hall Effect

This effect is a version of the Quantum Hall Effect, which describes an influence of gravitation in Hall resistance. This effect could be used to measure the value of the gravity constant.

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1 Quantum Hall Effect

The Quantum Hall Effect makes possible a very precisely measurement of resistance, because the Hall resistance only depends on nature constants. For derivation we consider an electrical conducting plate whith a thickness d, a width b and a length l. Lengthwise a direct current I_y is flowing through the plate. Perpendicularly the plate is penetrated by a magnetic field with a strength B_z. As a result the Lorentz force

$$K_L = \frac{lB_z I_y}{N}$$

is exerted on a single electron (N = number of all electrons in the plate). The electrons which are retained and exhausted respectively at the boundary cause an electric field E_H whose force

$$K_H = -eE_H \qquad (1.1)$$

to the electrons compensates the Lorentz force. The result for Hall voltage is

$$U_H = bE_H = -\frac{blB_z I_y}{Ne} .$$

Applies d≪l, d≪b to the plate dimensions there is a 2 dimensional electron gas. The number of electrons is

$$N = \frac{eblB_z}{h} i \qquad i = 1,2,3,...$$

(h = Planck's constant). The quantisation is a result of a circle movement of the electrons in the magnetic field (cyclotron resonance). Quantum mechanically allowed along these circle curves are only standing waves. Then the result for Hall voltage is

$$U_H = -R_H I_y$$

with the Hall resistance

$$R_H = \frac{h}{ie^2} .$$

2 Relativistic Quantum Hall Effect

To describe the relativistic Quantum Hall Effect equation (1.1) must be generalised. With the electromagnetic field tensor $F_{\mu\nu}$ the covariant shape of force is

$$K_\mu = -eF_{\mu\nu}u^\nu$$

where

$$\left(u^\nu\right) = \left(u^0, u^1, u^2, u^3\right)$$

is the 4-velocity. To speed of light applies the following relation:

$$c^2 = u_\nu u^\nu = g_{\mu\nu}u^\mu u^\nu .$$

To measure the Quantum Hall Effect in a gravity field, we consider a resting experiment facility e.g. on the surface of earth. Then we have

$$\left(u^\nu\right) = \left(u^0, 0, 0, 0\right)$$

and

$$c^2 = g_{00}\left(u^0\right)^2 .$$

The result for u^ν is:

$$\left(u^\nu\right) = \left(\frac{c}{\sqrt{g_{00}}}, 0, 0, 0\right).$$

Then we obtain

$$K_\mu = -eF_{\mu 0}u^0$$

or in a to (1.1) analogue component writing:

$$K_{\mathrm{H}} = -\frac{eE_{\mathrm{H}}}{\sqrt{g_{00}}} .$$

The value for g_{00} we get from Schwarzschild metrics

$$\left(g_{\mu\nu}\right) = \begin{pmatrix} 1-\frac{r_{\mathrm{s}}}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{r_{\mathrm{s}}}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

with the Schwarzschild radius

$$r_S = \frac{2GM}{c^2} .$$

The result is a modification of the value for R_H:

$$R'_H = R_H\sqrt{1 - \frac{r_S}{r}} \approx R_H(1 - \frac{GM}{rc^2}) .$$

For M and r the mass of earth and the earth radius have to be inserted. For $\frac{GM}{rc^2}$ we obtain a value of $7 \cdot 10^{-10}$. If we could increase the precision of measurements of the Quantum Hall Effect up to 10^{-12} the gravity constant G could be determined.

3 References

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